

Unit 5: Solving Quadratic Equations ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons. Show ALL work as is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

Solving Quadratic Equations by Factoring

1. Determine algebraically whether the given value is a solution for each quadratic equation.

a) $2x^2 - 5x - 3 = 0$, $x = 3$

$$2(3)^2 - 5(3) - 3 = 0$$

$$2 \times 9 - 15 - 3 = 0$$

$$18 - 15 - 3 = 0$$

$$0 = 0$$

LHS = RHS, hence $x = 3$ is solution

b) $3x^2 - 9 = 0$, $x = \sqrt{6}$

$$3x^2 - 9 = 0$$

$$3(\sqrt{6})^2 - 9 = 0$$

$$3 \times 6 - 9 = 0$$

$$18 - 9 = 0$$

$$9 \neq 0$$

LHS $\neq$ RHS.

hence $x = \sqrt{6}$
Not a solution.

2. Solve each quadratic equation.

a) $(x + 3)(x + 4) = 0$

$$(x + 3) = 0 \quad x + 4 = 0$$

$$x = -3$$

$$x = -4$$

$$x = -3, -4$$

b) $(x + 5)(3x - 4) = 0$

$$x + 5 = 0$$

$$3x - 4 = 0$$

$$x = -5 \quad ; \quad x = \frac{4}{3}$$

$$x = -5, \frac{4}{3}$$

c) $2(x + 4) = 0$

$$2(x + 4) = 0$$

$$x + 4 = 0$$

$$x = -4$$

d) $3x(x - 8) = 0$

$$3x(x - 8) = 0$$

$$x = 0$$

$$x - 8 = 0$$

$$x = 8$$

$$x = 0, 8$$

3. Solve by factoring.

a) $m^2 - 12m + 35 = 0$

$$m^2 - 7m - 5m + 35 = 0$$

$$m(m - 7) - 5(m - 7) = 0$$

$$(m - 5)(m - 7) = 0$$

$$m = 5, 7$$

b) $4c^2 - 49 = 0$

$$4c^2 = 49$$

$$c^2 = \frac{49}{4}$$

$$c = \sqrt{\frac{49}{4}} = \pm \frac{7}{2}$$

c) $y^2 - 11y = 0$

$$y^2 - 11y = 0$$

$$y(y-11) = 0$$

$$y = 0, 11$$

d) $2x^2 - 7x - 15 = 0$

$$2x^2 - 10x + 3x - 15 = 0$$

$$2x(x-5) + 3(x-5) = 0$$

$$(2x+3)(x-5) = 0$$

$$x = -\frac{3}{2}, 5$$

4. Solve by expanding, grouping like terms, and then factoring.

a) $m^2 + 8m = 3m + 24$

$$m^2 + 8m - 3m - 24 = 0$$

$$m(m+8) - 3(m+8) = 0$$

$$(m-3)(m+8) = 0$$

$$m = 3, -8$$

b) $x(x-6) = 2(x-8)$

$$x^2 - 6x = 2x - 16$$

$$x^2 - 6x - 2x + 16 = 0$$

$$x^2 - 8x + 16 = 0$$

$$x^2 - 4x - 4x + 16 = 0$$

$$x(x-4) - 4(x-4) = 0$$

$$(x-4)^2 = 0 \Rightarrow x = 4$$

c) $(2x-1)(x-3) = (x+1)(x-2)$

$$2x^2 - 6x - x + 3 = x^2 - 2x + x - 2$$

$$2x^2 - 7x + 3 = x^2 - x - 2$$

$$x^2 - 6x + 5 = 0$$

$$x^2 - 5x - x + 5 = 0$$

$$x(x-5) - 1(x-5) = 0$$

$$(x-1)(x-5) = 0$$

$$x = 1, 5$$

d) $(2p-1)^2 - 3 = (p-2)(p-1)$

$$4p^2 - 4p + 1 - 3 = p^2 - p - 2p + 2$$

$$3p^2 - p - 4 = 0$$

$$3p^2 + 3p - 4p - 4 = 0$$

$$3p(p+1) - 4(p+1) = 0$$

$$(3p-4)(p+1) = 0$$

$$p = \frac{4}{3}, -1$$

5. Write an expanded quadratic equation, with integer coefficients, from the given roots.

a) $x = -5, -3$

$$(x+5)(x+3) = 0$$

$$x^2 + 3x + 5x + 15 = 0$$

$$x^2 + 8x + 15 = 0$$

b) $x = \frac{1}{3}, \frac{2}{3}$

$$\left(x - \frac{1}{3}\right)\left(x - \frac{2}{3}\right) = 0$$

$$\frac{(3x-1)}{3} \cdot \frac{(3x-2)}{3} = 0$$

$$(3x-1)(3x-2) = 0$$

$$9x^2 - 6x - 3x + 2 = 0$$

$$9x^2 - 9x + 2 = 0$$

c) $x = 4, -\frac{1}{2}$

$$(x-4)(x+\frac{1}{2})=0$$

$$(x-4)(\frac{2x+1}{2})=0$$

$$(x-4)(2x+1)=0$$

$$2x^2+x-8x-4=0$$

$$2x^2-7x-4=0$$

d) $x = 0, \frac{4}{3}$

$$(x-0)(x-\frac{4}{3})=0$$

$$x(\frac{3x-4}{3})=0$$

$$x(3x-4)=0$$

$$3x^2-4x=0$$

6. Solve by factoring.

a) $x = \sqrt{3x-2}$

Sq. both sides

$$x^2 = 3x-2$$

$$x^2-3x+2=0$$

$$x^2-2x-x+2=0$$

$$x(x-2)-1(x-2)=0$$

$$(x-1)(x-2)=0$$

$$x=1, 2$$

b) $\sqrt{31-x} = x-1$

Sq. both sides

$$31-x = (x-1)^2$$

$$31-x = x^2-2x+1$$

$$x^2-x-30=0$$

$$x^2-6x+5x-30=0$$

$$x(x-6)+5(x-6)=0$$

$$(x+5)(x-6)=0$$

$$x=-5, 6$$

7. Find algebraically two consecutive integers with a product of 156.

Let the number be x .

Consecutive numbers will be

$$x(x+1)=156$$

$$x^2+x-156=0$$

$$x^2+13x-12x-156=0$$

$$x(x+13)-12(x+13)=0$$

$$(x-12)(x+13)=0$$

$$x=12, -13$$

Consecutive numbers

Consecutive number

When $x=12$

$$x+1=13$$

When $x=-13$

$$x+1=-12$$

8. Determine algebraically what number and its square differ by 30?

Let the number be x

$$x^2 - x = 30$$

$$x^2 - x - 30 = 0$$

$$x^2 - 6x + 5x - 30 = 0$$

$$x(x-6) + 5(x-6) = 0$$

$$(x+5)(x-6) = 0$$

$$x = -5, 6$$

9. The hypotenuse of a right triangle is 29cm. If the other two sides differ by 1cm, what are their lengths?

Let one side be x

Other side will be $x-1$

By Pythagoras theorem

$$x^2 + (x-1)^2 = (29)^2$$

$$x^2 + x^2 - 2x + 1 = 841$$

$$2x^2 - 2x + 1 = 841$$

$$2x^2 - 2x - 840 = 0$$

$$2(x^2 - x - 420) = 0$$

$$2(x^2 - 21x + 20x - 420) = 0$$

$$2[x(x-21) + 20(x-21)] = 0$$

$$2[(x+20)(x-21)] = 0$$

$$x \neq -20, \quad x = 21 \text{ cm}$$

Other side will be

$$x-1 = 21-1$$

$$= 20 \text{ cm}$$

10. When a football is kicked with a vertical speed of 20m/s, its height, h meters, after t seconds is approximated by the formula: $h = 20t - 5t^2$. How long after the kick is the football at a height of 15m?

We have to calculate t , at $h = 15\text{m}$

$$20t - 5t^2 = 15$$

$$5t^2 - 20t + 15 = 0$$

$$5t^2 - 5t - 15t + 15 = 0$$

$$5t(t-1) - 15(t-1) = 0$$

$$(5t-15)(t-1) = 0$$

$$5(t-3)(t-1) = 0$$

$$t = 3, 1$$

11. A rectangular garden has dimensions 6m by 8m. Leonard wants to build a path around the garden. He has 120m^2 of bricks. How wide is the pathway?

Let x be the width of the Pathway
 Length of rectangle $(6+2x)$; width of rectangle $(8+2x)$
 Area of rectangular garden = $6 \times 8 = 48$

$$(6+2x)(8+2x) - 48 = 120$$

$$28x + 4x^2 = 120$$

$$x^2 + 7x - 30 = 0$$

$$x^2 + 10x - 3x - 30 = 0$$

$$x(x+10) - 3(x+10) = 0$$

$$(x-3)(x+10) = 0$$

$$x = 3 ; x \neq -10 \text{ (Length can't be Negative)}$$

12. Solve by factoring.

a) $(2x+1)^2 - 3(2x+1) - 4 = 0$

$$4x^2 + 4x + 1 - 6x - 3 - 4 = 0$$

$$4x^2 - 2x - 6 = 0$$

$$2(2x^2 - x - 3) = 0$$

$$2[2x^2 + 2x - 3x - 3] = 0$$

$$2[2x(x+1) - 3(x+1)] = 0$$

$$2[(2x-3)(x+1)] = 0$$

$$x = \frac{3}{2}, -1$$

b) $3(x-1)^2 - 8(x-1) + 5 = 0$

$$3[x^2 - 2x + 1] - 8x + 8 + 5 = 0$$

$$3x^2 - 14x + 16 = 0$$

$$3x^2 - 6x - 8x + 16 = 0$$

$$3x(x-2) - 8(x-2) = 0$$

$$(3x-8)(x-2) = 0$$

$$x = \frac{8}{3}, 2$$

13. If one root is 5, find the value of k as well as the other root.

a) $x^2 - 3x + k = 0$

Put $x = 5$

$$(5)^2 - 3 \times 5 + k = 0$$

$$25 - 15 + k = 0$$

$$k = -10$$

$$x^2 - 3x - 10 = 0$$

$$x^2 - 5x + 2x - 10 = 0$$

$$x(x-5) + 2(x-5) = 0$$

$$(x+2)(x-5) = 0$$

$$x = -2, 5$$

b) $x^2 + kx + 40 = 0$

Put $x = 5$

$$(5)^2 + k(5) + 40 = 0$$

$$25 + 5k + 40 = 0 \Rightarrow k = -13$$

$$x^2 - 13x + 40 = 0$$

$$x^2 - 5x - 8x + 40 = 0$$

$$x(x-5) - 8(x-5) = 0$$

$$(x-8)(x-5) = 0$$

$$x = 8, 5$$

Lesson Two

Solving Quadratics by the Square Root Principle

1. Solve each equation exactly.

a) $x^2 + 12 = 48$

$$x^2 = 48 - 12$$

$$x^2 = 36$$

$$x = \pm 6$$

b) $2x^2 - 32 = 0$

$$2x^2 = 32$$

$$x^2 = 16$$

$$x = \pm 4$$

c) $m^2 - 7.5 = 92.5$

$$m^2 = 92.5 + 7.5$$

$$m^2 = 100$$

$$m = \pm 10$$

d) $3x^2 + 2 = 29$

$$3x^2 = 29 - 2$$

$$3x^2 = 27$$

$$x^2 = 9$$

$$x = \pm 3$$

e) $10 - z^2 = 4$

$$10 - 4 = z^2$$

$$6 = z^2$$

$$z = \pm \sqrt{6}$$

f) $-3 = 12 - 5d^2$

$$5d^2 = 12 + 3$$

$$5d^2 = 15$$

$$d^2 = 3$$

$$d = \pm \sqrt{3}$$

g) $(y + 2)^2 = 9$

$$(y + 2)^2 = 9$$

$$y + 2 = \pm 3$$

$$y + 2 = 3$$

$$y = 1$$

$$y + 2 = -3$$

$$y = -5$$

h) $(3 - c)^2 - 16 = 0$

$$(3 - c)^2 = 16$$

$$(3 - c) = \pm 4$$

$$3 - c = 4$$

$$c = -1$$

$$3 - c = -4$$

$$c = 7$$

i) $(m - \frac{1}{3})^2 = \frac{4}{9}$

$$(m - \frac{1}{3}) = \pm \frac{2}{3}$$

$$m - \frac{1}{3} = \frac{2}{3}$$

$$m = 1$$

$$m - \frac{1}{3} = -\frac{2}{3}$$

$$m = -\frac{1}{3}$$

j) $(1 - 2g)^2 = 49$

$$1 - 2g = \pm 7$$

$$1 - 2g = 7$$

$$1 - 7 = 2g$$

$$g = -3$$

$$1 - 2g = -7$$

$$1 + 7 = 2g$$

$$g = 4$$

k) $1.21 = (10a + 1)^2$

$$\pm 1.1 = (10a + 1)$$

$$10a = 1.1 - 1$$

$$10a = 0.1$$

$$a = 0.01$$

$$10a = -1.1 - 1$$

$$10a = -2.1$$

$$a = -0.21$$

l) $\frac{1}{3}(6x - 1)^2 - 3 = 0$

$$\frac{(6x - 1)^2 - 9}{3} = 0$$

$$(6x - 1)^2 - 9 = 0$$

$$(6x - 1) = \pm 3$$

$$6x - 1 = 3$$

$$x = \frac{2}{3}$$

$$6x - 1 = -3$$

$$x = -\frac{1}{3}$$

$$\begin{aligned} \text{m) } \frac{(a+2)^2}{4} &= 16 \\ (a+2)^2 &= 64 \\ (a+2) &= \pm 8 \\ a+2 &= 8 & a+2 &= -8 \\ a &= 6 & a &= -10 \end{aligned}$$

$$\begin{aligned} \text{n) } \left(a + \frac{3}{4}\right)^2 &= \frac{3}{16} \\ \left(a + \frac{3}{4}\right) &= \pm \sqrt{\frac{3}{16}} \\ a + \frac{3}{4} &= \pm \frac{\sqrt{3}}{4} \\ a &= \pm \frac{\sqrt{3} - 3}{4} \end{aligned}$$

$$\begin{aligned} \text{o) } ax^2 - b &= 0 \\ ax^2 - b &= 0 \\ ax^2 &= b \\ x &= \pm \sqrt{\frac{b}{a}} \end{aligned}$$

2. The height, h meters, of a falling object is related to the time, t seconds, the object has been falling by the formula $h = -4.9t^2 + d$ where d meters is the initial height of the object above the ground. The Bankers Hall building in Calgary is 196m tall. Express the time an object takes to reach the ground from this height as an exact number of seconds.

$$d = \text{Initial height} = 196 \text{ m}$$

$$h = \text{height} = 0 \text{ m}$$

$$0 = -4.9t^2 + 196$$

$$4.9t^2 = 196$$

$$t^2 = 40$$

$$t = \sqrt{40}$$

3. The volume of a cone with height, h (m), and radius r (m), is given by the formula $v = \frac{1}{3}\pi r^2 h$. What is the radius of a cone with volume 168cm^3 and height 9cm .

$$168 = \frac{1}{3} \times \pi \times r^2 \times 9$$

$$r^2 = \frac{56}{\pi}$$

$$r = \sqrt{\frac{56}{\pi}} = 4.22 \text{ m}$$

4. Five is added to an integer that is being quartered and then squared. This sum equals nine. Determine the integer algebraically.

$$\left(\frac{x}{4}\right)^2 + 5 = 9$$

$$\left(\frac{x}{4}\right)^2 = 4$$

$$\left(\frac{x}{4}\right) = \pm 2$$

$$x = \pm 8$$

5. The surface area of a cube is 120cm^2 . Determine the exact length of one of the edges.

$$6a^2 = 120$$

$$a^2 = 20$$

$$a = 2\sqrt{5}$$

6. Thirty less than the square of an integer is 139. Find the integer algebraically.

$$x^2 - 30 = 139$$

$$x^2 = 169$$

$$x = \pm 13$$

7. Solve each equation exactly.

a) $\frac{1}{x^2} = 16$

$$\frac{1}{16} = x^2$$

$$x = \pm \frac{1}{4}$$

b) $\frac{1}{x^2-1} = 5$

$$1 = 5(x^2-1)$$

$$5x^2 - 5 = 1$$

$$5x^2 = 6$$

$$x^2 = \frac{6}{5}$$

$$x = \pm \sqrt{\frac{6}{5}}$$

c) $\frac{7}{x+3} = x-3$

$$7 = (x-3)(x+3)$$

$$x^2 - 9 = 7$$

$$x^2 = 16$$

$$x = \pm 4$$

d) $\frac{4}{(x-1)^2} = \frac{9}{4}$

$$(x-1)^2 = \frac{4 \times 4}{9}$$

$$(x-1)^2 = \frac{16}{9}$$

$$(x-1)^2 = \pm \frac{4}{3}$$

$$x = 1 \pm \frac{4}{3}$$

$$x = \frac{7}{3}, -\frac{1}{3}$$

Lesson Three

Solving Quadratics Using the Quadratic Formula

1. Solve using the quadratic formula. Give exact answers as integers, fractions, or simplest radical form.

a) $2x^2 - 5x + 2 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - (4 \times 2 \times 2)}}{2 \times 2}$$

$$x = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$$

$$x = 2, \frac{1}{2}$$

b) $3n^2 - 11n - 14 = 0$

$$n = \frac{-(-11) \pm \sqrt{(-11)^2 - (4 \times 3 \times (-14))}}{2 \times 3}$$

$$n = \frac{11 \pm \sqrt{289}}{6} = \frac{11 \pm 17}{6}$$

$$n = \frac{14}{3}, -1$$

c) $f^2 - 6f + 4 = 0$

$$f = \frac{-(-6) \pm \sqrt{(-6)^2 - (4 \times 1 \times 4)}}{2 \times 1}$$

$$= \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$$

$$f = 3 + \sqrt{5}, 3 - \sqrt{5}$$

d) $x^2 + 7x + 3 = 0$

$$x = \frac{-7 \pm \sqrt{(7)^2 - (4 \times 1 \times 3)}}{2 \times 1}$$

$$= \frac{-7 \pm \sqrt{37}}{2}$$

e) $6t^2 - t - 1 = 0$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - (4 \times 6 \times (-1))}}{2 \times 6}$$

$$= \frac{1 \pm \sqrt{25}}{12} = \frac{1 \pm 5}{12}$$

$$t = \frac{1}{2}, -\frac{1}{3}$$

f) $5h^2 + 7h + 2 = 0$

$$h = \frac{-7 \pm \sqrt{(7)^2 - (4 \times 5 \times 2)}}{2 \times 5}$$

$$= \frac{-7 \pm \sqrt{9}}{10} = \frac{-7 \pm 3}{10}$$

$$h = -\frac{2}{5}, -1$$

2. Solve exactly using the quadratic formula.

a) $25q^2 + 70q + 49 = 0$

$$q = \frac{-70 \pm \sqrt{(70)^2 - (4 \times 25 \times 49)}}{2 \times 25}$$

$$= \frac{-70 \pm \sqrt{0}}{50} = -\frac{7}{5}$$

$$q = -\frac{7}{5}$$

b) $12v^2 - 192 = 0$

$$v = \frac{-0 \pm \sqrt{0^2 - (4 \times 12 \times (-192))}}{2 \times 12}$$

$$= \pm \frac{\sqrt{9216}}{24} = \pm \frac{96}{24}$$

$$v = \pm 4$$

c) $3r^2 - 4r = 0$

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 3 \times 0)}}{2 \times 3}$$

$$= \frac{4 \pm \sqrt{16}}{6} = \frac{4 \pm 4}{6}$$

$$r = \frac{4}{3}, 0$$

d) $5x^2 + 12x + 9 = 0$

$$x = \frac{-12 \pm \sqrt{(12)^2 - (4 \times 5 \times 9)}}{2 \times 5}$$

$$x = \frac{-12 \pm \sqrt{-36}}{10}$$

Hence No real roots

3. Solve using the quadratic formula. Express answers to the nearest hundredth.

a) $0.2z^2 - z - 3.2 = 0$

Multiplying both sides by 10

$$2z^2 - 10z - 32 = 0$$

$$z = \frac{-(-10) \pm \sqrt{(-10)^2 - (4 \times 2 \times (-32))}}{2 \times 2}$$

$$= \frac{10 \pm \sqrt{84}}{4}$$

$$= \frac{5 \pm \sqrt{84}}{2}$$

$$z = -2.22; 7.22$$

b) $1.2x^2 = 1.4x - 1$

Multiplying both sides by 10

$$12x^2 = 14x - 10$$

$$12x^2 - 14x + 10 = 0$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - (4 \times 12 \times 10)}}{2 \times 12}$$

$$= \frac{14 \pm \sqrt{-284}}{24}$$

Hence No real roots

4. Solve using the quadratic formula. Answer in simplest radical form.

a) $2x^2 + 6x + 2 = 0$

$$x = \frac{-6 \pm \sqrt{(6)^2 - (4 \times 2 \times 2)}}{2 \times 2}$$

$$= \frac{-6 \pm 2\sqrt{5}}{4}$$

$$= \frac{-3 \pm \sqrt{5}}{2}$$

b) $z^2 - 6z + 7 = 0$

$$z = \frac{-(-6) \pm \sqrt{(-6)^2 - (4 \times 1 \times 7)}}{2 \times 1}$$

$$= \frac{6 \pm 2\sqrt{2}}{2}$$

$$z = 3 \pm \sqrt{2}$$

5. Solve using the quadratic formula.

a) $(n - 4)(n - 2) = 12$

$$n^2 - 6n + 8 = 12$$

$$n^2 - 6n - 4 = 0$$

$$n = \frac{-(-6) \pm \sqrt{(-6)^2 - (4 \times 1 \times (-4))}}{2 \times 1}$$

$$= \frac{6 \pm 2\sqrt{13}}{2}$$

$$n = 3 \pm \sqrt{13}$$

b) $(2x - 1)(3x + 5) = (x + 2)(2x - 1)$

$$(2x - 1)(3x + 5) - (x + 2)(2x - 1) = 0$$

$$(2x - 1)[3x + 5 - x - 2] = 0$$

$$(2x - 1)(2x + 3) = 0$$

$$4x^2 + 4x - 3 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - (4 \times 4 \times (-3))}}{2 \times 4}$$

$$= \frac{-4 \pm 8}{8}$$

$$x = \frac{1}{2}, -\frac{3}{2}$$

$$c) 3m^2 - (5m + 1)(2m - 3) = 3$$

$$3m^2 - 10m^2 + 13m + 3 = 3$$

$$-7m^2 + 13m = 0$$

$$m = \frac{-13 \pm \sqrt{(13)^2 - (4 \times (-7) \times 0)}}{2 \times (-7)}$$

$$= \frac{-13 \pm 13}{-14}$$

$$m = 0, \frac{13}{7}$$

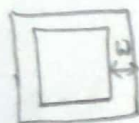
$$d) 2(w - 2)(w + 1) - (w + 3) = 0$$

$$2w^2 - 2w - 4 - w - 3 = 0$$

$$2w^2 - 3w - 7 = 0$$

$$w = \frac{-(-3) \pm \sqrt{(-3)^2 - (4 \times 2 \times (-7))}}{2 \times 2}$$

$$w = \frac{3 \pm \sqrt{65}}{4}$$



6. A photograph measuring 16cm by 12cm is to be surrounded by a mat before framing. The width of the mat is to be the same on all sides of the photograph. The area of the mat is equal to the area of the photograph. Find the width of the mat, to the nearest tenth of a centimeter.

$$\text{Area of frame} = (16 + 2w)(12 + 2w) - (16 \times 12)$$

$$\text{Area of Photo} = 16 \times 12$$

$$(16 + 2w)(12 + 2w) - (16 \times 12) = (16 \times 12)$$

$$4w^2 + 56w + 192 = 384$$

$$4w^2 + 56w - 192 = 0$$

$$w = \frac{-56 \pm \sqrt{56^2 - 4 \times 4 \times (-192)}}{2 \times 4} = \frac{-56 \pm \sqrt{97}}{4}$$

$$w = 2.85 ; w \neq -16.85$$

7. The hypotenuse of a right triangle measures 20cm. The sum of the lengths of the other two sides is 28cm. Find the lengths of these two sides.

Let the length of one side be x

other side = $28 - x$

$$x^2 + (28 - x)^2 = (20)^2$$

$$2x^2 - 56x + 784 = 400$$

$$2x^2 - 56x + 384 = 0$$

$$x = \frac{-(-56) \pm \sqrt{56^2 - 4 \times 2 \times 384}}{2 \times 2} = 16, 12$$

8. Elli travelled 700km by truck to deliver some ostriches to her friend's farm, she then returned the same distance by car. The car averaged 10km/h faster than the truck and took 1 hour less time. What was the speed of the car?

v be the speed of truck

For truck $vt = 700$ - ①

For car $(v+10)(t-1) = 700$

$vt - v + 10t - 10 = 700$ - ②

From eq ① + ②

$vt - v + 10t - 10 = vt$

$v = 700/t$ - ③

Substituting

$10t - \frac{700}{t} - 10 = 0$

$t^2 - t - 70 = 0$

$t = \frac{-(-1) \pm \sqrt{(-1)^2 - (4 \times 1 \times (-70))}}{2 \times 1}$

$t = \frac{1 \pm \sqrt{281}}{2} = 8.88; t \neq -7.88$

9. When two pumps are turned on they can fill the pond at Helmut's farm in seven hours. Individually, one pump takes two hours longer than the other to fill the pond. Find the time to fill the pond by the slower pump by itself.

$\frac{7}{x} + \frac{7}{x+2} = 1$

$7(x+2) + 7x = x(x+2)$

$7x + 14 + 7x = x^2 + 2x$

$x^2 - 12x - 14 = 0$

$x = \frac{-2 \pm 10\sqrt{2}}{2}$

$x = 6 \pm 5\sqrt{2}$

$x = 13.07; x \neq -1.07$

10. A boat travelled 30km upstream, against the current, to deliver some feed from Elli, to Helmut's farm, it then turned around and headed 35km downstream to dock for the night. If the speed of the current was 3km/h and the upstream trip took 45min longer than the downstream trip, what was the speed of the boat?

Net Speed upwards = $x - 3$

Net Speed downwards = $x + 3$

Time upwards = $30/(x-3)$

Time downwards = $35/(x+3)$

45 min = 0.75 hours

$\frac{30}{x-3} = \frac{35}{x+3} + 0.75$

Solving

$0.75x^2 + 5x - 201.75 = 0$

$x = 13.4 \text{ km/h}$

Lesson Four

The Discriminant

1. Calculate the value of the discriminant and then state the nature of the roots.

a) $x^2 + 11x + 24 = 0$

$$D = \sqrt{(11)^2 - (4 \times 1 \times 24)}$$

$$= 25 > 0$$

Hence two real roots.

b) $n^2 - 4n + 2 = 0$

$$D = \sqrt{(-4)^2 - (4 \times 1 \times 2)}$$

$$= 8 > 0$$

Hence two real roots

c) $4m^2 - 20m + 25 = 0$

$$D = \sqrt{(-20)^2 - (4 \times 4 \times 25)}$$

$$= 0$$

Hence one root

d) $2x^2 - 5x + 8 = 0$

$$D = \sqrt{(-5)^2 - (4 \times 2 \times 8)}$$

$$= -39 < 0$$

No real roots

e) $\sqrt{5}x^2 + 7x + 2\sqrt{5} = 0$

$$D = (7)^2 - 4 \times \sqrt{5} \times 2\sqrt{5}$$

$$= 9 > 0$$

Hence two real roots

f) $\frac{x-1}{2} - x^2 = 3$

$$\frac{x-1-2x^2}{2} = 3$$

$$-2x^2 + x - 7 = 0$$

$$D = (1)^2 - 4 \times (-2) \times (-7)$$

$$= -55 < 0 \quad \text{No real roots}$$

2. Determine the value(s) of k that give the type of solution indicated.

a) $x^2 - 6x + k = 0$; one real root

One real root $D = 0$

$$D = (-6)^2 - 4k = 0$$

$$36 = 4k$$

$$k = 9$$

b) $kx^2 - 2x + 1 = 0$; two real roots

Two real roots $D > 0$

$$D = (-2)^2 - 4k > 0$$

$$4k < 4$$

$$k < 1$$

c) $x^2 + 4x - 2k = 0$; no real roots

No real root $D < 0$

$$D = (4)^2 - 4 \times 1 \times (-2k) < 0$$

$$16 + 8k < 0$$

$$k < -2$$

d) $2kx^2 - 4x + 3 = 0$; two real roots

Two real root $D > 0$

$$D = (-4)^2 - 4 \times (2k) \times 3 > 0$$

$$16 - 24k > 0$$

$$k < \frac{2}{3}$$

e) $m^2 + 4km + 1 = 0$; one real root

One real root $D = 0$

$$D = (4k)^2 - (4 \times 1) = 0$$

$$16k^2 = 4$$

$$k = \pm \frac{1}{2}$$

f) $(k+1)x^2 - 2x - 3 = 0$; no real roots

No real root $D < 0$

$$D = (-2)^2 - 4(k+1)(-3) < 0$$

$$= 4 + 12k + 12 < 0$$

$$k < -\frac{4}{3}$$